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Average vertical stress over a stratum beneath uniformly loaded square/circular footings

Ankurjyoti Saikia[1] , Binanda K Narzary[2], Jayanta D Bharali[3]
[1](Department of Civil Engineering, Tezpur University, Assam, India)

Abstract: *Based on 2:1 load dispersion method, an analytical expression is derived in this study that enables to compute the average vertical stress over a layer beneath the center of uniformly loaded square footings. The solution is presented in a dimensionless graphical form for allowing easier computations. The solution can also be applied in case of circular footings by replacing the circular footing by a square footing of equivalent area. Present solution does not require any point-stress equation or design chart and avoids multi-stage calculations. This solution may find immense application in estimating maximum settlement below uniformly loaded square/circular footings.*

Keywords - *vertical stress, non-dimensional, footing, settlement*

I. Introduction:

Over the years, a good deal of analytical and graphical solutions have been developed for computing vertical stresses at a point within the half-space under loaded areas of uniform or non-uniform geometries. Boussinesq's stress distribution formula [1] under a point load, Newmark's [2] simplified solution for estimating stresses under a uniformly loaded rectangular area, graphical solution of Fadum [3] for computing stresses below a uniformly loaded rectangular area are examples of such solutions.

Few other solutions of this category are: Newmark's influence chart [4] for computing vertical stress below uniformly loaded areas of regular or irregular geometry, Design chart proposed by Foster and Ahlvin [5] for estimating vertical stress under uniform circular loads, numerical influence values contributed by Ahlvin and Ulery [6] for determining patterns of stresses, strains, and deflections in a homogeneous half-space below uniform circular load etc.

All these equations and design charts are deduced for point-stress calculations and cannot be applied directly to compute average vertical stress over a layer. The difficulty in determining average vertical stress lies in its parabolic variation with depth. The average stress is mostly calculated by the conventional mid-depth approach that gives low values of average stress and consequently gives underestimated settlements.

Other than the mid-depth practice, few statistical approaches are extensively used for determining average vertical stress over a layer. But these require the aid of elastic point-stress equations or dimensionless design charts. Moreover, these approaches involve stages of calculations as the stresses are to be determined at successive depths within the layer.

In view of the above, a generalized expression is deduced in this study that enables to estimate the average vertical stress over a layer under uniformly loaded square footings. The solution can also be used in case of circular footings. The solution is depicted in a non-dimensional graphical form to facilitate easy computations.

II. Background :

Estimation of average vertical stress over a layer is required in predicting the consolidation settlement of compressible layers under loaded footings. Few statistical approaches are adopted for fair estimation of average vertical stress. The most common of these approaches is the arithmetic average of vertical stresses. By determining the vertical stresses at successive depths within a layer, the average vertical stresses can be obtained from the arithmetic mean of stresses.

An alternate solution to this problem is the use of harmonic mean of vertical stresses [7]. By the use of harmonic mean, more weight-age is given to the upper layers as they are susceptible to more settlements. Another solution is the use of Sympson's rule by determining the stresses at three locations: at top, mid-depth, and bottom of a layer [1]. Making use of Sympson's rule, average stress can be determined with reasonable accuracy and in fewer steps than the other two approaches. However, all these are indirect statistical techniques that require the aid of elastic point-stress equations or design charts and involve repetitive calculations.

Author [8] deduced a generalized solution for computing average vertical stress under uniformly distributed circular load based on the elastic equation for stress distribution due to a circular load.

III. Analytical Solution:

A layer of thickness (H) who's top and bottom surfaces are at depths H_1 and H_2 from the footing base is referenced for deducing the expression. The integration scheme is shown in Fig. 1.

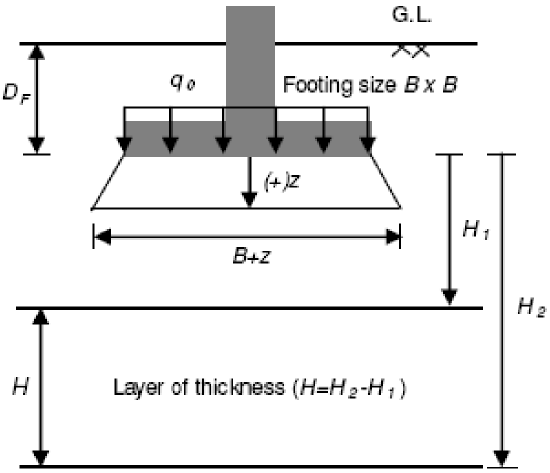

Figure 1: Adopted scheme (modified after Saikia, 2012)

Let q_0 be the stress acting at the base of a square footing of width, B. Using 2:1 load dispersion method, the vertical stress at a depth, z below the point of origin of stress is given by:

$$\sigma_z = q_0 \frac{B^2}{(B+z)^2} \qquad (1)$$

Average vertical stress can be obtained by suitably integrating the above expression over the entire thickness of layer:

$$\bar{\sigma}_z = \frac{\int_{H_1}^{H_2} q_0 \frac{B^2}{(B+z)^2} dz}{\int_{H_1}^{H_2} dz}$$

$$\bar{\sigma}_z = \frac{q_0 B^2}{(H_2 - H_1)} \int_{H_1}^{H_2} \frac{dz}{(B+z)^2}$$

Substituting, $(B+z) = u$ and on changing the limits of integration, above equation reduces to the following form:

$$\bar{\sigma}_z = \frac{q_0 B^2}{(H_2 - H_1)} \int_{B+H_1}^{B+H_2} \frac{du}{u^2}$$

$$\bar{\sigma}_z = \frac{q_0 B^2}{(H_2 - H_1)} \left(\frac{1}{B+H_1} - \frac{1}{B+H_2} \right)$$

$$\bar{\sigma}_z = \frac{q_0 B^2}{(B+H_1)(B+H_2)} \qquad (2)$$

The above equation represents the average vertical stress over a layer of thickness ($H = H_2 - H_1$). The average vertical stress can be expressed in a non-dimensional form in terms of influence factor ($I_{av} = \bar{\sigma}_z / q_0$) by rearranging the terms of Eq. (2):

$$\frac{\bar{\sigma}_z}{q_0} = \frac{(H_2/H_1)}{(H_2/H_1 + H_2/B)(1 + H_2/B)} \qquad (3)$$

The average vertical stress influence values are depicted in the form of a non-dimensional design chart by plotting the variation of the same against H_2/B for different values of H_2/H_1 in Fig. 2.

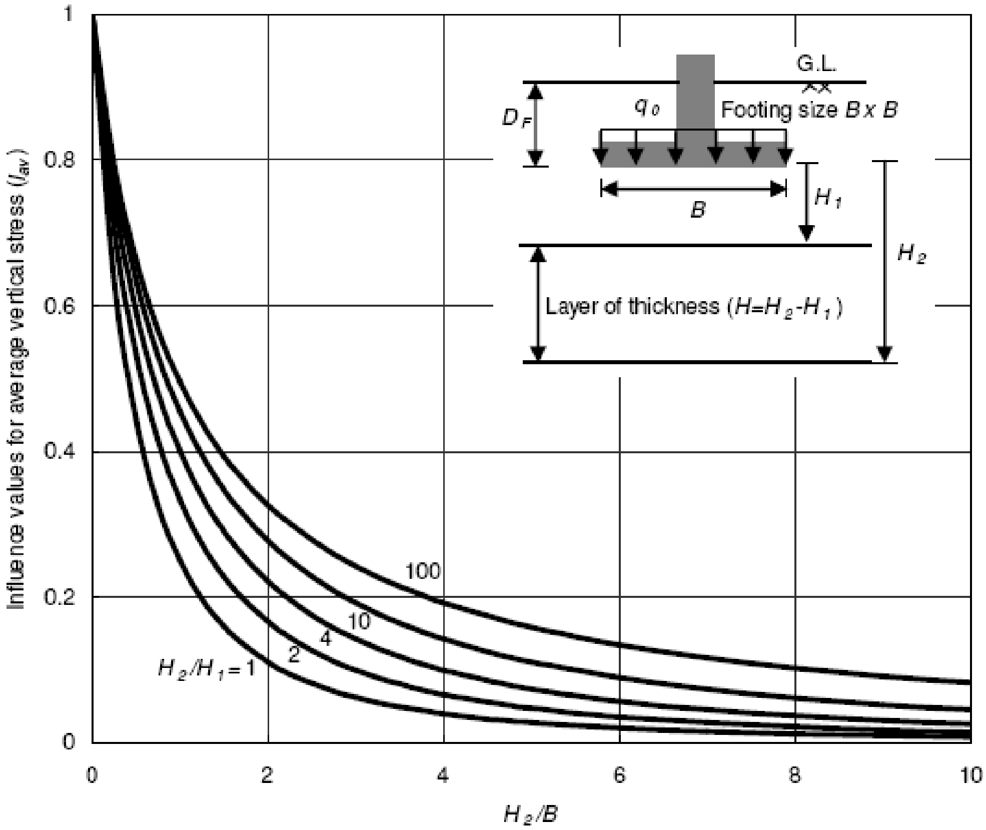

Figure 2: Design chart for average vertical stress under uniformly loaded square footings

With reference to Fig. 2, $H_2/H_1 = 1$, implies a layer of infinitesimal thickness. The vertical stress itself is the average in such cases. It can also be concluded that when the compressible layer extends to a depth exceeding ten times the width of a footing, i.e. $H_2 \geq 10B$, the average vertical stress influence values are negligible, if the layer is thin in particular (for smaller values of H_2/H_1). The average vertical stress over a layer in such cases is of minor significance in settlement analysis.

The solution can also be applied in case of circular footings by replacing the circular footing by a square footing of equivalent area.

IV. Conclusion:

The solution proposed in this study is based on the 2:1 load dispersion method. Undervalued results are anticipated if the layer lies at depths less than about 1.5 times the width of a footing. Because in such cases, 2:1 load dispersion method gives lower stress values than the exact elastic solutions.

The solution is primarily deduced for square footings but is applicable for circular footings too. In case of circular footings, square footing of equivalent area should be used.

Unlike the conventional multi-stage calculation techniques, the average vertical stress can be obtained directly using Eq. (2). Alternately, the design chart depicted in Fig. 2 can be used as a ready reference for calculating the same. This solution may find immense application in predicting consolidation settlement of compressible soils overlain by uniformly loaded square/circular footings.

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